Generalized-Material-Independent PML Absorbers Used for the FDTD Simulation of Electromagnetic Waves in 3-D Arbitrary Anisotropic Dielectric and Magnetic Media

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***Abstract*—By introducing the material-independent quantities (electric displacement D and flux density B) into the finite-difference time-domain (FDTD) model, a generalized-material-independent perfectly matched layer (GMIPML) absorber used to absorb electromagnetic waves propagating in three-dimensional (3-D) general anisotropic dielectric and magnetic media is proposed. Within the proposed GMIPML absorber, D and B are directly absorbed, whereas E and H are simultaneously absorbed through the relations between E and D, as well as H and B. It is shown that with the help of this GMIPML absorber, Berenger's perfectly matched layer (PML) absorbing boundary condition (ABC) can be simply and effectively extended to 3-D arbitrary anisotropic materials consisting of both arbitrary permittivity and permeability tensors.**

***Index Terms*—FDTD, general anisotropic media, GMIPML absorber.**

I. Introduction

When the finite-difference time-domain (FDTD) method is applied to open problems, developing accurate and efficient absorbing boundary conditions (ABC's) for the problems is obviously one of the most important issues. Recently, Berenger's perfectly matched layer (PML) ABC [1] has been widely accepted as one of the best ABC's for the truncation of FDTD lattices, and its capability has been further enhanced by extending it to three-dimensional (3-D) isotropic [2]–[5], diagonal anisotropic [6]–[8], and general anisotropic [9], [10] media. In particular, the approaches proposed in [2]–[10] are based on the $\mathbf{E}$ and $\mathbf{H}$ formulations and, inside the PML absorbers, the electric conductivity σ^E and magnetic conductivity σ^H are involved. Using the PML absorber [2]–[8] based on the $\mathbf{E}$ and $\mathbf{H}$ formulations does not produce any trouble if the material under consideration is isotropic or diagonally anisotropic. However, for arbitrary anisotropic media (especially when both permittivity and permeability tensors occur), the PML matching condition [9], [10] is quite complicated and, hence, implementation of this PML is rather difficult and cumbersome. This is mainly due to the fact that the conductivities σ^E and σ^H used in the PML absorbers are *direct* functions of the tensors $[\varepsilon]$ and/or $[\mu]$ of the materials under consideration.

Alternatively, very recent investigations [11]–[13] indicate that Berenger's PML can be easily extended to anisotropic cases if the material-independent perfectly matched layer (MIPML) absorbers [11]–[13] are adopted. Numerical results in [11] and [12] demonstrated that by introducing $\mathbf{D}$ (or $\mathbf{B}$) into the FDTD model, electromagnetic waves propagating in two-dimensional (2-D) arbitrary anisotropic dielectric [11] (or magnetic [12]) media can be simply and effectively absorbed. In this paper, following the works presented in [11]–[13], a generalized-material-independent perfectly matched layer (GMIPML) absorber that can be applied to 3-D general anisotropic media consisting of both arbitrary permittivity and permeability tensors is proposed. In particular, within the proposed GMIPML absorber, the conductivities σ^D and σ^B (instead of σ^E and σ^H) are used. Once such an arrangement on the conductivity is made, $\mathbf{D}$ and $\mathbf{B}$ are directly absorbed by the GMIPML absorber, whereas $\mathbf{E}$ and $\mathbf{H}$ can be simultaneously absorbed through the relation $\mathbf{E} = (1/\varepsilon_0)[\varepsilon]^{-1}\mathbf{D}$ and $\mathbf{H} = (1/\mu_0)[\mu]^{-1}\mathbf{B}$. As a result, the GMIPML absorber proposed in this paper is totally independent of both dielectric and magnetic anisotropies of the material and, therefore, Berenger's PML can be simply and easily extended to 3-D arbitrary anisotropic media consisting of both permittivity and permeability tensors.

Manuscript received September 16, 1997.

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Publisher Item Identifier S 0018-9480(98)07232-9.

II. Theory

Assume that the relative permittivity and permeability tensors of an arbitrary anisotropic (loss-free) medium are $[\varepsilon]$ and $[\mu]$, respectively. Generally speaking, hybrid waves (i.e., none of the six components of $\mathbf{E}$ and $\mathbf{H}$ is zero) will propagate in the above anisotropic medium. If a variant of Yee's unit cell (i.e., the components of the $\mathbf{D}$ (or $\mathbf{B}$) field are, respectively, located at the same positions as the components of the $\mathbf{E}$ (or $\mathbf{H}$) field) is used, then the hybrid wave propagating in the GMIPML absorber (noting that only $\mathbf{D}$ and $\mathbf{B}$ need to be split) is described by

$$\frac{\partial D_{xy}}{\partial t} + \sigma_y^D D_{xy} = \frac{\partial H_z}{\partial y}$$
$$\frac{\partial D_{xz}}{\partial t} + \sigma_z^D D_{xz} = -\frac{\partial H_y}{\partial z} \tag{1a}$$
$$\frac{\partial D_{yz}}{\partial t} + \sigma_z^D D_{yz} = \frac{\partial H_x}{\partial z}$$
$$\frac{\partial D_{yx}}{\partial t} + \sigma_x^D D_{yx} = -\frac{\partial H_z}{\partial x} \tag{1b}$$
$$\frac{\partial D_{zx}}{\partial t} + \sigma_x^D D_{zx} = \frac{\partial H_y}{\partial x}$$
$$\frac{\partial D_{zy}}{\partial t} + \sigma_y^D D_{zy} = -\frac{\partial H_x}{\partial y} \tag{1c}$$
$$\frac{\partial B_{xy}}{\partial t} + \sigma_y^B B_{xy} = -\frac{\partial E_z}{\partial y}$$
$$\frac{\partial B_{xz}}{\partial t} + \sigma_z^B B_{xz} = \frac{\partial E_y}{\partial z} \tag{1d}$$
$$\frac{\partial B_{yz}}{\partial t} + \sigma_z^B B_{yz} = -\frac{\partial E_x}{\partial y}$$
$$\frac{\partial B_{yx}}{\partial t} + \sigma_x^B B_{yx} = \frac{\partial E_z}{\partial x} \tag{1e}$$
$$\frac{\partial B_{zx}}{\partial t} + \sigma_x^B B_{zx} = -\frac{\partial E_y}{\partial x}$$
$$\frac{\partial B_{zy}}{\partial t} + \sigma_y^B B_{zy} = -\frac{\partial E_x}{\partial y} \tag{1f}$$

where the parameters σ_x^D, σ_y^D, σ_z^D, σ_x^B, σ_y^B, and σ_z^B (which relate to $\mathbf{D}$ and $\mathbf{B}$, respectively) are the conductivities used in the proposed GMIPML absorber, and the matching conditions are

$$\sigma_p^D = \sigma_p^B, \qquad \text{where } p = x, y, z. \tag{2}$$

From the above equation, one can see that unlike the approaches [9], [10] based on $\mathbf{E}$ and $\mathbf{H}$ formulations, the form of the matching condition of the proposed MIPML is very simple and not functions of $[\varepsilon]$ and $[\mu]$. In addition, it should be noted that unlike the electric conductivity [1], ε_0 is not involved in the expression of the theoretical reflection factor $R(\theta)$ for the electric displacement conductivity $\sigma^D(\rho)$. Most especially, $R(\theta)$ used for $\sigma^D(\rho)$ in the GMIPML absorber is

$$R(\theta) = e^{-2(\cos\theta/c)\int_0^\delta \sigma^D(\rho)\,d\rho}. \tag{3}$$

Since within the GMIPML absorbers, $\mathbf{D}$ and $\mathbf{B}$ (instead of $\mathbf{E}$ and $\mathbf{H}$) are directly absorbed, and $\mathbf{E}$ and $\mathbf{H}$ are not split, updating equations used for $\mathbf{D}$ and $\mathbf{B}$ are slightly different from those used in the original Berenger's scheme for $\mathbf{E}$ and $\mathbf{H}$. For example, updating equations for D_{zx} and B_{zx} in the GMIPML absorber should read, respectively, as

$$D_{zx}^{n+1}(i,j) = e^{-\sigma_x^D(i)\Delta t} D_{zx}^{n}(i,j) + \frac{\left(1 - e^{-\sigma_x^D(i)\Delta t}\right)}{\sigma_x^D(i)\Delta x} \cdot \left[H_y^{n+0.5}(i+0.5,j) - H_y^{n+0.5}(i-0.5,j)\right] \quad \text{(4a)}$$

and

$$\begin{aligned} B_{zx}^{n+0.5}&(i+0.5, j+0.5) \\ &= e^{-\sigma_x^D(i+0.5)\Delta t} B_{zx}^{n-0.5}(i+0.5, j+0.5) \\ &\quad - \frac{\left(1 - e^{-\sigma_x^B(i+0.5)\Delta t}\right)}{\sigma_x^B(i+0.5)\Delta x}\left[E_y^n(i, j+0.5) - E_y^n(i+1, j+0.5)\right]. \end{aligned} \quad \text{(4b)}$$

From the above discussion, one can see that once the conductivities σ^E and σ^H are replaced by the conductivities σ^D and σ^B, the proposed GMIPML (for both the interfaces and corner regions) can simply be constructed in a similar manner as that used in the 3-D Berenger's PML for isotropic media [2]. In addition, $\mathbf{E}$ (or $\mathbf{H}$) are updated from the relationship $\mathbf{E} = (1/\varepsilon_0)[\varepsilon]^{-1}D$ (or $\mathbf{H} = (1/\mu_0)[\mu]^{-1}\mathbf{B}$), combining with an averaging approximation procedure on the components of the $\mathbf{D}$ (or $\mathbf{B}$) field in space.

III. Numerical Results

To validate and confirm the proposed GMIPML absorber, hybrid waves propagating in a 3-D uniaxial anisotropic medium consisting of both dielectric ($\varepsilon_1 = 2$ and $\varepsilon_2 = 3$) and magnetic ($\mu_1 = 2$ and $\mu_2 = 3$) materials are studied. In particular, if θ is the angle between the optical axis (within the XZ-plane) and the X-direction, then the nonzero elements of the permittivity tensor $[\varepsilon]$ are

$$\varepsilon_{xx} = \varepsilon_1 \cos^2 \theta + \varepsilon_2 \sin^2 \theta \quad \text{(5a)}$$

$$\varepsilon_{yy} = \varepsilon_1 \quad \text{(5b)}$$

$$\varepsilon_{zz} = \varepsilon_1 \sin^2 \theta + \varepsilon_2 \cos^2 \theta \quad \text{(5c)}$$

$$\varepsilon_{xz} = \varepsilon_{zx} = (\varepsilon_2 - \varepsilon_1) \sin\theta \cos\theta \quad \text{(5d)}$$

whereas the nonzero elements of the permeability tensor $[\mu]$ are fixed as $\mu_{xx} = 2.25$, $\mu_{yy} = 2.75$, $\mu_{zz} = 2.0$, and $\mu_{xy} = \mu_{yx} = 0.433$. For most of the simulations, the following parameters are used: the total computational domain (including the GMIPML) is 40 × 40 × 41, the space steps are $\Delta x = \Delta y = \Delta z = 6$ mm, the time step is $\Delta t = 12.5$ ps, the number of the GMIPML cells (in all the X-, Y-, and Z-directions) is $N = 10$, and a cubic (i.e., $m = 3$) distribution of the conductivity $\sigma^D(\rho)$ with a normal theoretical reflection $R(0) = 0.001\%$ in the GMIPML is used. The system is excited by D_z with a smooth compact pulse [11] located at the center of the domain, and all results are recorded at 120 time steps. To clearly demonstrate how the waves (for the cases $\theta = 0$, $\pi/6$, $\pi/4$, $\pi/3$, and $\pi/2$, respectively) are absorbed, the reference FDTD solution (calculated with a much larger computational domain) and the solution obtained with the GMIPML absorber for the E_z-field component along a special line (X, 11, 20.5) (i.e., the interface between the anisotropic medium and the GMIPML absorber) are shown in Fig. 1. It can be seen from Fig. 1 that within the anisotropic medium (i.e., within the range $11\Delta x \leq X \leq 31\Delta x$), no significant difference between these two solutions is observed. To further evaluate the performance of the proposed GMIPML absorber,

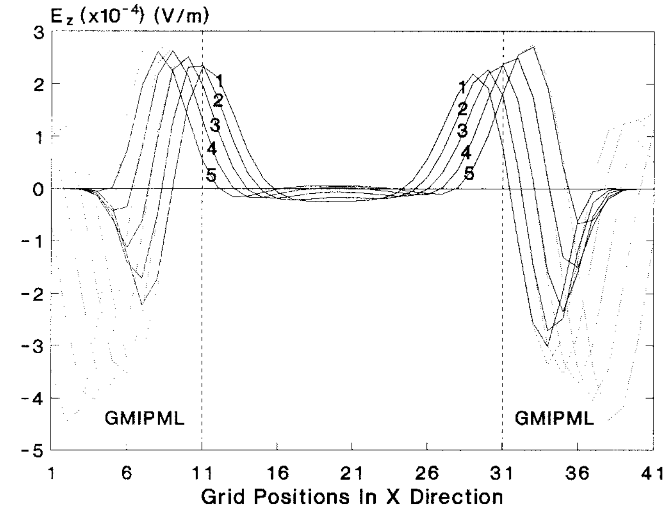

Fig. 1. The reference FDTD solution (dotted curves) and the solution obtained with the GMIPML (solid curves), for the E_z-field component of the waves along the line (X, 11, 20.5), where curves 1–5 are for $\theta = 0$, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$, respectively.

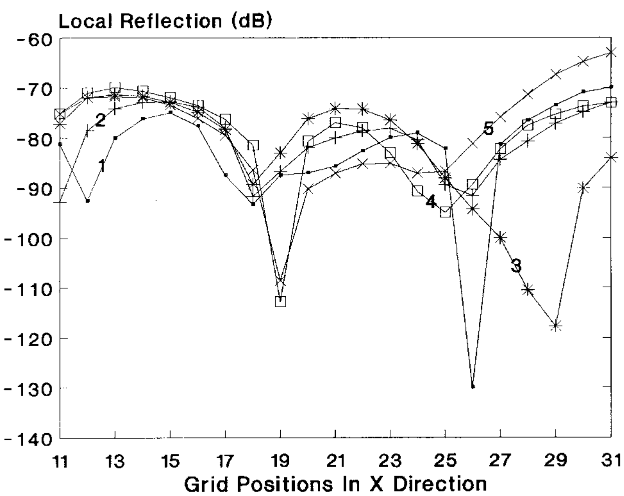

Fig. 2. Normalized local reflections for the E_z-field component of the waves along the line (X, 11, 20.5), where curves 1–5 are for $\theta = 0$, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$, respectively.

Fig. 2 shows the normalized (with respect to the maximum value of the reference solution within the region $11\Delta x \leq X \leq 31\Delta x$) local reflections (for the cases $\theta = 0$, $\pi/6$, $\pi/4$, $\pi/3$, and $\pi/2$, respectively) caused by the GMIPML for the E_z-field component along the interface. The results in Fig. 2 indicate that the proposed GMIPML performs quite well (better than 60 dB) for all the different anisotropic cases. It is not surprising that due to the anisotropy, slightly different absorbing performances are observed for different anisotropic cases since the same parameters in the GMIPML absorber were used for different values of θ.

In addition, as expected, the absorbing performance will be changed (or enhanced) while the parameters (such as N, $R(0)$, and m) used in the GMIPML absorber vary. Numerical investigations were carried out for the cases when the number of the GMIPML cells (N) and the grading order of the GMIPML (m) are varied. Fig. 3 shows how the normalized local reflections for the E_z-field component of the wave ($\theta = \pi/6$) along the line (X, 11, 20.5) are influenced by varying the parameters N and m of the GMIPML

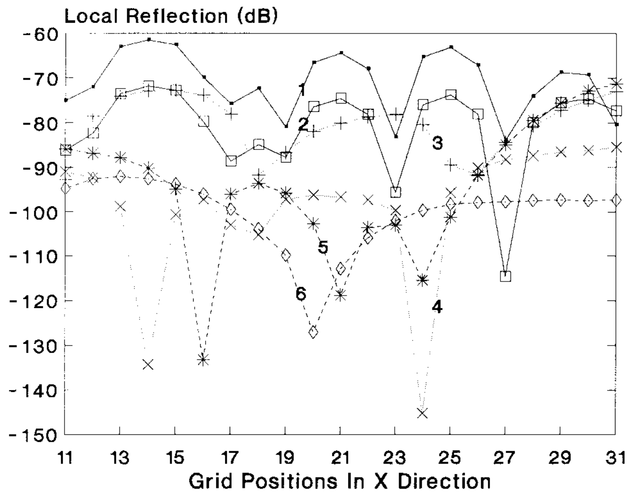

Fig. 3. Normalized local reflections for the E_z-field component of the wave ($\theta = \pi/6$) along the line (X, 11, 20.5), where Curve 1: $N = 10$, $m = 2$, Curve 2: $N = 15$, $m = 2$, Curve 3: $N = 10$, $m = 3$, Curve 4: $N = 15$, $m = 3$, Curve 5: $N = 10$, $m = 4$, and Curve 6: $N = 15$, $m = 4$.

absorber. Noting that when $N = 15$, the total computational domain is simultaneously changed to $50 \times 50 \times 51$, and the X-axis of Fig. 3 is given according to the case $N = 10$. It can be seen from Fig. 3 that by adjusting the parameters used in the GMIPML absorber, better absorbing performance can be achieved.

IV. Conclusions

By introducing the material-independent quantities $\mathbf{D}$ and $\mathbf{B}$ into the FDTD model, a GMIPML absorber aimed for absorbing electromagnetic waves propagating in 3-D arbitrary anisotropic media consisting of both permittivity and permeability tensors is developed. In contrast with the previous PML absorbers, the conductivities σ^D and σ^B, instead of σ^E and σ^H, are used. The main reason of using σ^D and σ^B in the proposed absorber is due to the fact that these conductivities are independent of the anisotropy of the material. As a consequence, Berenger's PML can be simply and effectively extended to 3-D arbitrary anisotropic materials. Furthermore, due to the special feature (i.e., the material independence) of the proposed GMIPML absorber, it can also be used to absorb electromagnetic waves propagating in materials consisting of loss, dispersion, and nonlinearity [14] with slight modifications. Furthermore, unsplit-field formulations (i.e., without splitting the $\mathbf{D}$ and $\mathbf{B}$ fields) of the GMIPML absorber have been recently proposed [15] by the author.

References

[1] J. P. Berenger, "A perfectly matched layer for the absorption of electromagnetic waves," *J. Comput. Phys.*, vol. 114, pp. 185–200, 1994.

[2] D. S. Katz, E. T. Thiele, and A. Taflove, "Validation and extension to three dimensions of the Berenger PML absorbing boundary condition for FD-TD meshes," *IEEE Microwave Guided Wave Lett.*, vol. 4, pp. 268–230, Aug. 1994.

[3] W. C. Chew and W. H. Weedon, "A 3-D perfectly matched medium from modified Maxwell's equations with stretched coordinates," *Microwave Opt. Technol. Lett.*, vol. 7, pp. 599–604, 1994.

[4] R. Mittra and U. Pekel, "A new look at the perfectly matched layer (PML) concept for the reflectionless absorption of electromagnetic waves," *IEEE Microwave Guided Wave Lett.*, vol. 3, pp. 84–86, Mar. 1995.

[5] J. Fang and Z. Wu, "Generalized perfectly matched layer for absorption of propagating and evanescent waves in lossless and lossy media," *IEEE Trans. Microwave Theory Tech.*, vol. 44, pp. 2216–2222, Dec. 1996.

[6] Z. S. Sacks, D. M. Kingsland, R. Lee, and J. F. Lee, "A perfectly matched anisotropic absorber for use as an absorbing boundary condition," *IEEE Trans. Antennas Propagat.*, vol. 43, pp. 1460–1463, Dec. 1995.

[7] L. Zhao and A. C. Cangellaris, "GT-PML: Generalized theory of perfectly matched layers and its application to the reflectionless truncation of finite-difference time-domain grids," *IEEE Trans. Microwave Theory Tech.*, vol. 44, pp. 2555–2563, Dec. 1996.

[8] S. D. Gedney, "An anisotropic perfectly matched layer-absorbing medium for the truncation of FDTD lattices," *IEEE Trans. Antennas Propagat.*, vol. 44, pp. 1630–1639, Dec. 1996.

[9] B. Chen, D. G. Fang, and C. Gao, "Modified PML for the general anisotropic media," *J. Nanjing Univ. Sci. Technol.*, vol. 21, pp. 453–456, 1997.

[10] I. V. Perez, S. G. Garcia, R. G. Martin, and B. G. Olmedo, "Extension of Berenger's absorbing boundary conditions to match dielectric anisotropic media," *IEEE Microwave Guided Wave Lett.*, vol. 7, pp. 302–304, Sept. 1997.

[11] A. P. Zhao, J. Juntunen, and A. V. Raisanen, "Material independent PML absorbers for arbitrary anisotropic dielectric media," *Electron. Lett.*, vol. 33. pp. 1535–1536, 1997.

[12] A. P. Zhao and A. V. Raisanen, "Extension of Berenger's PML absorbing boundary conditions to arbitrary anisotropic magnetic media," *IEEE Microwave Guided Wave Lett.*, vol. 8, pp. 52–54, Jan. 1998.

[13] A. P. Zhao and M. A. Rinne, "Theoretical proof of the material independent PML absorbers used for arbitrary anisotropic media," *Electron. Lett.*, vol. 34, pp. 48–49, 1998.

[14] A. P. Zhao, "Application of the material-independent PML absorbers to the FDTD analysis of electromagnetic waves in nonlinear media," *Microwave Opt. Technol. Lett.*, vol. 17, pp. 164–168, 1998.

[15] ——, "Unsplit-field formulations for generalized material independent PML absorbers," in *1998 IEEE MTT-S Microwave Symp. Dig.* pp. 469–472.

Accurate Analysis of Periodic Structures with an Additional Symmetry in the Unit Cell from Classical Matrix Eigenvalues

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***Abstract*—Dispersion diagrams of periodic structures with an additional symmetry in the unit cell are investigated by the example of a parallel-plate waveguide loaded with irises of zero thickness. The propagation constants of the Floquet modes are determined from the *classical* eigenvalues of a non-Hermitian matrix.**

***Index Terms*—Eigenfunctions, eigenvalues, integral equations, periodic structures.**

I. Introduction

The growing interest in photonic bandgap (PBG) materials has created a demand for efficient methods of analysis of periodic structures [1]. Periodic structures have been the subject of numerous

Manuscript received February 20, 1998.

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Publisher Item Identifier S 0018-9480(98)07231-7.